Passive Velocity Filtering for Haptic Applications with Wave Control

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Abstract—This paper introduces a passive velocity filtering technique for multirate haptic applications with wave variable control. In the proposed method, velocity is estimated via Euler’s method and is filtered in the wave domain using a filter bank-like structure placed in the communications. The paper derives the passivity condition for the wave domain communications with the filter bank, and uses it to set up a minimax optimization which yields the velocity filter. Frequency domain analysis of haptic interaction in a slowly updated virtual environment predicts that the proposed wave domain velocity filter increases the transparency of the interaction. Experiments in which a Novint Falcon haptic device touches a slowly updated virtual wall show significantly less noise in the force feedback provided to the user.

I. INTRODUCTION

Digital velocity estimation from position samples has been a challenging task in various applications. Several methods are available for velocity computation, each suitable for a specific application. In-depth reviews of these methods are presented in [1], [2], [9].

In haptics, the velocity of the haptic device is used to compute the force of the virtual environment (VE) and the force feedback provided to the user. Therefore, an ideal velocity estimator: (i) is accurate and thus, enhances the transparency of the interaction; (ii) adds no lag to the force feedback loop and thus, does not endanger stability; and (iii) is noise-free and thus, introduces unperceivable noise in the forces applied to the user.

Wave domain communications [10] are widely used in teleoperation and haptics because they are robust to constant communication delay even when the communications transmission rate or the VE update rate are lower than the update rate of the force feedback loop [14]. Velocity estimation is more critical for systems with wave communications than for systems with power domain communications because of the local feedback loop at the operator. This loop has gain equal to the wave impedance. For transparent interaction in low frequencies, the wave impedance is typically selected large [11]. However, a large wave impedance amplifies the velocity noise and degrades transparency in high frequencies.

To the best of our knowledge, velocity estimation and filtering for haptic systems with wave communications was not considered in prior research. Existing haptics velocity estimators fall into two categories. One category includes estimation techniques which demand: (i) a haptic device outfitted with additional hardware compared to a commercially available device [3], [5], [6], [12], [17]; or (ii) an accurate device model which is not practical to obtain [7], [16]. Another category includes velocity estimators which require only software implementation [8], [15]. The effect of the adaptive windowing velocity estimator [8] on the stability and the transparency of haptic interaction is unclear. The Finite Impulse Response differentiator [15] enhances the passivity of the haptic system, but does not eliminate noise from the forces applied to the user.

This paper introduces a new wave domain filter bank-like structure (Figure 1) which can be used to passively filter velocity in haptic applications with passive multirate wave communications [13], [14]. Note that the system depicted in Figure 1 is a multirate system. The master side including the haptic interface is updated fast and the slave side including the VE is updated slowly. The rate change happens in the communication channels which is modeled by downsampler/upsampler. The output and input waves are related to the velocities and forces at the haptic interface (master) and VE (slave) sides via [11]:

\[
\begin{align*}
    u_m(t) &= \frac{F_m(t) + b x_m(t)}{2b} \\
v_m(t) &= -\frac{F_m(t) + b x_m(t)}{2b} \\
v_s(t) &= \frac{-F_s(t) + b x_s(t)}{2b} \\
u_s(t) &= \frac{-F_s(t) + b x_s(t)}{2b}
\end{align*}
\]

where \( \dot{x}_m \) is the velocity of the haptic interface; \( \dot{x}_s \) is the velocity command transmitted to the VE through wave variable communications; \( F_m \) is the VE force; \( F_m \) is the force applied to the haptic interface by the wave controller; \( u_m \) and \( v_m \) are the output waves; \( u_s \) and \( v_s \) are the input waves; and \( b \) is the wave impedance.

As shown in Figure 1, the proposed filter bank comprises a low-pass filter \( LP_1 \) and a high-pass filter \( HP \). It splits the outgoing wave signal \( u_m \) into a low frequency component \( u_{LP} \) which it feeds to the VE, and a high frequency component \( v_{HP} \) which it uses to filter the estimated velocity of the haptic interface \( \dot{x}_m \). The role of \( LP_1 \) was discussed in [13], [14]: (i) it removes noise; (ii) it reduces wave reflections; and (iii) it prevents aliasing and thus, guarantees passive multirate wave communications when the network transmission rate is limited or the VE is updated slowly. Assuming the network or the VE sampling interval to be an integer multiple of the sampling time of the force control loop \( T \), \( M \) is the communications downsampling/upsampling factor (Figure 1),
i.e. the rate change ratio. The role of HP is to passively filter velocity to: (i) increase the transparency of the haptic interaction in high frequencies; (ii) eliminate noise from the forces applied to users; and (iii) not endanger the stability of the haptic system. These objectives can be achieved because HP is a wave filter and the passivity of wave communications is unaffected by phase lag and time delay [11].

As explained in Section II, the filter bank in Figure 1 is implemented using the equivalent architecture shown in Figure 2. To account for the limitations of practical filters, the design of LP and is formulated as a minimax problem. The optimization criterion is supplied by the passivity condition of the multirate wave communications with the filter bank. This passivity condition is derived by considering the energy balance in the communication channels. Frequency domain analysis and experiments demonstrate that the velocity filtering method proposed in this paper improves the transparency of the haptic system, especially in high frequencies, and removes noise for the forces rendered to the user.

In the remainder of this paper, Section II introduces the problem associated with the passive multirate wave control. Section III discusses the passivity of the proposed architecture including velocity filtering. Section IV presents the proposed filter design methodology. Section V and Section VI investigate the performance of passive multirate wave variable control with the proposed passive velocity via frequency domain analysis and experiments, respectively. Section VII summarizes the conclusions drawn from this work.

II. PROBLEM DEFINITION

For an impedance haptic interface with wave control the feedback force is computed as:

\[ F_m = b\ddot{x}_m - \sqrt{2b}v_m \]  \hspace{1cm} (2)

If the wave impedance is large, the term \( \ddot{x}_m \) becomes dominant in high frequencies leading to a large feedback force. Moreover, the force feedback applied to the user becomes too noisy. Decreasing the wave impedance would alleviate these difficulties, but would also decrease the transparency of the haptic interaction in low frequencies. Velocity low pass filtering would also improve transparency and denoise the force feedback in high frequencies. However, filtering in the power domain would impose lag which may destabilize the haptic system. A velocity observer would eliminate the phase lag, but would demand an accurate model of the haptic interface. Such a model is generally impractical to obtain for consumer-grade haptic devices because of inherent nonlinearities and variation of physical properties from one device to the next.

For haptic applications with multirate wave variable control, this work proposes a passive velocity filtering method which can be designed independent of the haptic interface and the wave impedance. Consider \( v_{HP} \), the output of the HP in Figure 1, \( v_{HP} = u_{HP} \). Since \( u_m \) is computed via:

\[ u_m = \sqrt{2b}\ddot{x}_m - v_m \]  \hspace{1cm} (3)

and \( v_m \) includes mostly low and medium frequencies, high frequencies are introduced in \( v_{HP} \) mainly by \( \ddot{x}_m \). Therefore, after feeding back half of \( v_{HP} \) and some manipulation, Equation (2) can be written in the frequency domain as:

\[ F_m(z) = b\ddot{x}_m(z)(1 - HP(z)) - \sqrt{2b}v_m(z), \] \hspace{1cm} (4)

where \( (1 - HP(z)) \) is effectively a low pass filter which denoises the velocity estimate \( \ddot{x}_m \). Equation (4) also proves that the filter bank-like structure is Figure 1 is equivalent.

![Figure 1. Wave domain filter bank-like structure for passive velocity filtering for haptic applications with passive multirate wave communications.](image)

![Figure 2. Master (haptic device) side of the haptic system, with filters LP and LP1 which are equivalent to the filter bank HP and LP1 in Figure 1.](image)
to the filter structure in Figure 2 provided \( LP_V(z) = (1 - HP(z)) \). The effect of \( LP_V \) on the high frequency performance of a haptic interaction system with passive multirate wave communications is investigated next.

A. Frequency Domain Analysis

In [13], [14], we showed that aliasing should be prevented in order to guarantee the passivity of multirate wave communications, and proposed to low pass filter the outgoing wave \( u_m \) before downsampling it using a filter \( LP_1 \) with cutoff frequency less than half the update frequency of the VE. The performance cost imposed by \( LP_1 \) is investigated in this section through frequency domain analysis.

The multirate system shown in Figure 1 but without HP is lifted [4] to a unirate system whose transmitted admittance:

\[
H(z) = \frac{X_m(z)}{F_b(z)},
\]

is computed for various wave impedances where \( F_b \) is the hand input and by considering that the update rate of the master side is \( T = 1 \) Khz, VE sampling interval is \( T_{VE} = 0.02 \) s and the VE stiffness and damping are \( K_{VE} = 2000 \) N/m and \( B_{VE} = 10 \) Ns/m, respectively. The frequency response of the transmitted admittance in Equation (5) is compared with the frequency response of the ideal transmitted admittance, obtained by directly coupling the haptic interface to a VE with \( T_{VE} = 0.001 \) s, \( K_{VE} = 2000 \) N/m and \( B_{VE} = 10 \) Ns/m, respectively.

Figure 3 plots the frequency response of the transmitted admittance in low frequencies and for different wave impedances. In this frequency range, when the hand input can be considered constant, increasing the wave impedance leads to a transmitted admittance that is closer to the ideal transmitted admittance. This is expected because \( LP_1 \) behaves like a virtual coupler with stiffness [11]:

\[
K_{\text{filter}} = 2b\lambda,
\]

for constant hand input and in steady state. In Equation (6), \( \lambda \) is the cutoff frequency of \( LP_1 \). Hence, the transparency of low frequency interaction increases as the wave impedance increases.

On the other hand, a large wave impedance degrades transparency at high frequencies, as illustrated in Figure 4. Note in this figure that increased wave impedance, similarly to added damping, decreases the first natural frequency of the haptic system. The transparency of the multirate wave communications could be increased by increasing the cutoff frequency of \( LP_1 \) at the expense of stability [14].

To improve high frequency performance and remove noise from the force feedback without compromising stability, this paper introduces the filter bank-like structure in Figure 1. Because this structure filters velocity in the wave domain, the phase lag it introduces does not affect the passivity of wave communications [11].

Section III derives the passivity condition for the multirate wave communications with the filter bank-like structure, and Section IV uses this condition to design HP.

III. Passivity Analysis

The passivity condition for the left (master or haptic device) side of the haptic system in Figure 1 is:

\[
\Delta E(t) = \frac{1}{2} \left[ \sum_{k=0}^{N} u_{m_k}^2(k) \cdot T - \sum_{k=0}^{N} v_{m_k}^2(k) \cdot T \right] \geq 0,
\]

\( \forall t \geq 0, \) (7)

where \( u_{m_k} \) and \( v_{m_k} \) are truncated signals defined via:

\[
u_{\phi}(\tau) = \begin{cases} 
0 & \text{if } \tau < 0 \\
u(\tau) & \text{if } 0 \leq \tau \leq \theta \\
0 & \text{if } \tau > \theta
\end{cases},
\]

and \( T \) is the sampling time of the force control loop. After substitution from Equation (8), Equation (7) becomes:

\[
\Delta E(t) = \frac{1}{2} \left[ \sum_{k=0}^{\infty} u_{m_k}^2(k) \cdot T - \sum_{k=0}^{\infty} v_{m_k}^2(k) \cdot T \right] \geq 0,
\]

\( \forall t \geq 0. \) (9)

Using Parseval’s theorem, the right hand side in Equation (9) can be written:

\[
\frac{1}{2} \left[ \frac{TM}{2\pi} \int_{-\pi}^{\pi} |U_m(e^{j\omega})|^2 - |V_m(e^{j\omega})|^2 d\omega \right],
\]

where \( U_m \) and \( V_m \) are the discrete-time Fourier transforms of \( u_m \) and \( v_m \) respectively. Since:

\[
v_m = v_{LP} + v_{HP},
\]
where \( v_{LP} \) and \( v_{HP} \) are the outputs of the VE and HP, respectively, Equation (10) can be written as:

\[
\frac{1}{2} \left( \frac{TM}{2\pi} \int_{-\pi}^{\pi} |U_m(e^{j\omega})|^2 - \left| V_{LP}(e^{j\omega}) + V_{HP}(e^{j\omega}) \right|^2 d\omega \right), \tag{12}
\]

Assuming a passive VE, Equation (12) becomes:

\[
\frac{1}{2} \left( \frac{TM}{2\pi} \int_{-\pi}^{\pi} |U_m(e^{j\omega})|^2 \left[ 1 - \left| H_{LP}(e^{j\omega}) \right|^2 \right] + \frac{|H_{HP}(e^{j\omega})|^2}{4} + 2\text{Re}\left[ \frac{H_{LP}(e^{j\omega})H_{HP}^*(e^{j\omega})}{2} d\omega \right] \right), \tag{13}
\]

where \( H_{HP}^* \) denotes complex conjugate of \( H_{HP} \), and \( \text{Re}(\cdot) \) denoted the real part of a complex number. Finally, the passivity condition for the multirate wave communications with the filter bank can be written as:

\[
1 > \left[ \left| H_{LP}(e^{j\omega}) \right|^2 + \frac{|H_{HP}(e^{j\omega})|^2}{4} \right] + 2\text{Re}\left[ \frac{H_{LP}(e^{j\omega})H_{HP}^*(e^{j\omega})}{2} \right], \tag{14}
\]

Note that, Equation (14) is independent of VE and haptic interface, provided they are passive. If LP and HP are ideal, Equation (14) is always satisfied. For practical filters, Equation (14) provides the filter design criterion. This criterion is used in the following section to formulate a minimax optimization which yields the coefficients of HP.

IV. FILTER BANK DESIGN

This section illustrates the design of the two filters LP\(_1\) and HP of the proposed filter bank-like structure considering haptic interaction with a VE updated at a rate of 50 Hz. As discussed in [14], LP\(_1\) has cutoff frequency less than half the update rate of the VE to ensure the passivity of the multirate wave communications. Therefore, LP\(_1\) is taken as a first order filter with transfer function:

\[
H_{LP}(z) = \frac{0.05}{z - 0.95} \tag{15}
\]

and its sampling time is equal to the update rate of the master side, i.e. 1 Khz.

Selecting a first order HP with transfer function:

\[
H_{HP}(z) = \frac{z - 1}{z - \alpha} \tag{16}
\]

\( \alpha \) can be computed by solving a constrained minimax optimization. The optimization minimizes the error function obtained by evaluating the right hand side of Equation (14) at 512 frequency points equally distributed in the \([0, 500]\) Hz interval. The constraint is defined so as to guarantee the stability of HP and to prevent overlapping between the bandwidths of LP\(_1\) and HP, i.e., \( 0 < \alpha < 0.95 \). The Matlab Optimization toolbox provides the HP optimal parameter \( \alpha = 0.92913 \).

V. FREQUENCY DOMAIN ANALYSIS

The admittance transmitted to the user by the proposed filter bank-like structure is shown in Figure 5 for high frequencies\(^1\). This figure demonstrates that the high frequency performance of the new architecture is much closer to that of the ideal interaction, considered to be achieved when the user is directly coupled to the VE and the VE is updated at the same rate as the force control loop. Hence, the new architecture permits larger wave impedances to be used which, in turn, improve the transmitted admittance in the low frequency range.

VI. EXPERIMENTS

This section validates the analytical results via two controlled experimental interactions with a slow virtual wall through a Novint Falcon device. The haptic device is connected to a personal computer running Windows Vista on Intel Core 2 Duo CPU at 2.67GHz with 2 GB RAM. A one degree of freedom VE comprising a virtual wall with stiffness \( K_{VE} = 2000 \) N/m and damping \( B_{VE} = 10 \) Ns/m runs as a C++ console application on the same computer. The Novint Falcon API is used to run the force control loop and the proposed filtering at 1 kHz, and to update the VE at 50 Hz. The wave impedance is \( b = 30 \) Ns/m.

The first experiment aims to illustrate the performance of the proposed velocity filtering method in terms of noise attenuation. In this experiment, a sinusoidal hand input with amplitude \( F_h = 1 \) N and frequency 5 Hz is applied to the haptic interface via commands sent to the motors. The force feedback provided to the user is shown in Figure 6, Figure 7 and Figure 8 for haptic interaction without velocity

\(^1\) The transmitted impedance in low frequencies is not shown because velocity filtering changes it little unless the wave impedance is increased.
filtering, with a velocity filter designed without considering the passivity condition Equation (14) ($\alpha = 0.95$), and with the passive velocity filtering proposed in this paper ($\alpha = 0.929$), respectively. These figures demonstrate that: (i) the proposed wave domain velocity filtering provides noise-free force feedback to users connected to a slow VE via multirate wave communications; and (ii) the interaction can become unstable in practical implementation unless the velocity filter is designed to maintain the passivity of the communications.

The second experiment investigates the transient performance of haptic interaction with the slow VE when the user applies a step force $F_h = 2$ N. Figure 9 and Figure 10 depict the results for passive multirate wave communications without and with the proposed velocity filtering, respectively. Note that, in addition to removing the noise from the force feedback, passive wave domain velocity filtering reduces almost by a factor of 2 the peak transient force rendered to the user. This reduction validates the improved performance in high frequency predicted in Section V.

VII. CONCLUSIONS

To improve the performance of passive multirate wave control of haptic interaction across networks with limited transmission rate or within slowly updated VEs, this paper has introduced a filter bank-like structure for passive filtering of device velocity. The filters of the new structure have been
designed to maintain the passivity of the communications. Both analysis and experimental results have been presented to demonstrate the advantages of the proposed filter bank:

- in high frequencies, it increases transparency as measured through the admittance transmitted to the user;
- it can filter noisy velocity input passively;
- its performance can be quantitatively evaluated and improved although the filters are designed based on passivity considerations;
- its filters are designed independent of the device and the VE, provided they are passive;
- the filter design is a unirate problem.

The main limitation of the proposed filter bank is the requirement that the network transmission rate or the VE update rate be fixed. Future work will seek to extend passive velocity filtering to haptic interaction with VEs with variable update rate. Performance improvement through integrating the filter bank-like structure with a local model of interaction will also be investigated.

VIII. ACKNOWLEDGMENTS

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REFERENCES


